Calculus BC
Summer Review Problems

Name: _____
Date: _____ Pd: _____

Work neatly! If I can't read it, I won't grade it. Show all work on a separate sheet of paper, and "box" your final answer! You may only use a calculator when indicated.

1. Find the following limits.

   a. $\lim_{x\to 4}\dfrac{x^2-3x-4}{x^2-6x+8}$

   b. $\lim_{x\to 0}\dfrac{3x}{\sin 4x}$

   c. $\lim_{x\to\infty}\dfrac{\sqrt{x+5}+2}{9x-1}$

   d. $\lim_{x\to\frac{\pi}{2}^+}(\tan x)$

   e. $\lim_{x\to\infty}\dfrac{2+x+5x^2}{3x^2-1}$

2. Use the definition of the derivative to find $f'(x)$ where $f(x)=\dfrac{1}{x+3}$.

3. Find $\dfrac{dy}{dx}$ for the following functions. You do not need to simplify your answer.

   a. $y=5x^2-\sqrt{2x}+\cos x$

   b. $y=\left(\ln\left(x^3+1\right)\right)\left(\tan\left(4-2x\right)\right)$

   c. $y=\sin^3\left(2x+e^x\right)$

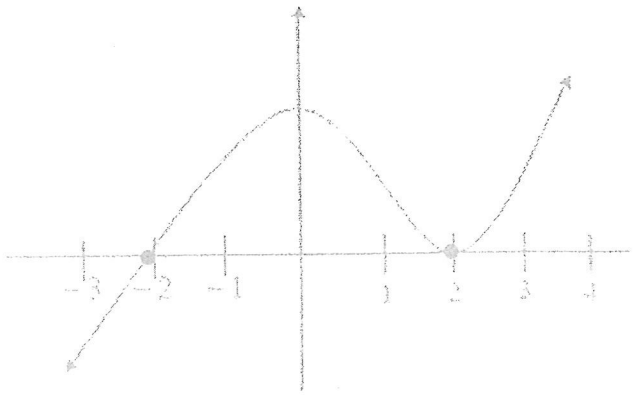
   d. $y=\dfrac{x^2-7x+1}{5x^3-x}$ (you needn't expand)

   e. $y=\int_x^4\dfrac{dt}{1+t^3}$

   f. $y=x^{\cos x}$

4. Let $y(x)$ be defined implicitly by $x^3y^2-4e^x+y=9$. Find $\dfrac{dy}{dx}$.

5. Suppose that the derivative $f'(x)$ of a function is given by the graph:



   a. Find the open intervals where f is increasing and those where f is decreasing.
   b. Find the open intervals where f is concave up and those where f is concave down.
   c. If $f(0)=2$, draw a plausible graph of f.

6. Determine an equation of the tangent line to the curve of $f(x)=2e^{3x}$ at the point where the curve crosses the line $y=2$.

7. A man 6 feet tall walks at a rate of 5 feet per second toward a light that is 20 feet above the ground. When he is 10 feet from the base of the light, at what rate is the tip of his shadow moving?

8. An airplane is flying in still air with an airspeed of 240 miles per hour. If it is climbing at an angle of 22°, find the rate at which it is gaining altitude. (CALCULATOR PERMITTED)

9. A rectangle is bounded by the x- and y-axes and the graph of $y=\frac{6-x}{2}$. What length and width should the rectangle have so that its area is a maximum?

10. Find the absolute maximum and absolute minimum of $f(x)=\frac{4}{3}x^3-6x^2+8x+10$ on $[0, 3]$. Give both x and y coordinates.

11. Evaluate the following indefinite integrals.

a. $\int\frac{5}{x\ln x}dx$

b. $\int\csc^2 x e^{\cot x}dx$

c. $\int\left(e^{2x}+6x-\sqrt[5]{x}\right)dx$

d. $\int\frac{3x}{\sqrt{1-x}}dx$

e. $\int\left(x^3-6\right)^2dx$

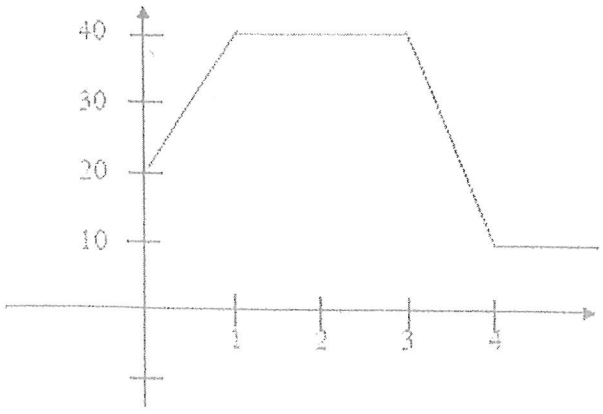
12. Evaluate the following definite integrals.

a. $\int_0^3\left(x^2e^{x^3}\right)dx$

b. $\int_0^{(\pi/4)^2}\frac{\sec^2\sqrt{x}}{\sqrt{x}}dx$

13. Find the area of the region between the graphs of $y=x^4$ and $y=x+1$. (CALCULATOR PERMITTED)

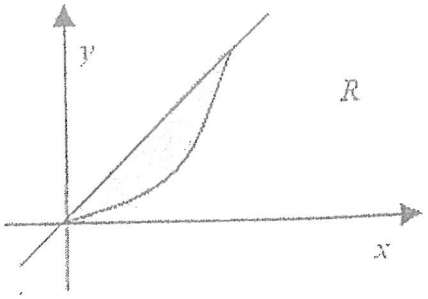
14. Let the velocity of a particle be given by the following graph.



a. Sketch the acceleration.

b. What is the total distance traveled between $t=1$ and $t=4$?

15. Let R be the region bounded by the curves $y = x^2$ and $y = 4x$. (CALCULATOR PERMITTED)



a. Find the volume of the solid obtained by revolving this region around the line $y = -1$.
b. Find the volume of the solid obtained by revolving this region around the line $x = -1$.

16. The temperature of a metal rod, 7 meters long, is $5x$ (in °C) at a distance x meters from one end of the rod. What is the average temperature of the rod?

17. Find the length of the curve $y = \frac{x^4}{4} + \frac{1}{8x^2}$ on the interval [-3, -1]. (CALCULATOR PERMITTED)

18. Find the length of the curve $y = 2\arctan x$ on the interval [0, 3]. (CALCULATOR PERMITTED)

19. Find the area of the surface obtained by rotating the curve $y^2 = 9x + 9$ on the interval [0, 6] about the x-axis. (CALCULATOR PERMITTED)

20. Find the area of the surface obtained by rotating the curve $y = 4 - x^2$ on the interval [0, 2] about the y-axis. (CALCULATOR PERMITTED)